# DIRICHLET-NEUMANN SHAPE OPTIMIZATION PROBLEMS

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We consider spectral optimization problems of the form

$$\min\Big\{\lambda_1(\Omega;D):\ \Omega\subset D,\ |\Omega|=1\Big\},$$

where $D$ is a given subset of the Euclidean space $\mathbb{R}^d$. Here $\lambda_1(\Omega;D)$ is the first eigenvalue of the Laplace operator $-\Delta$ with Dirichlet conditions on $\partial\Omega\cap D$ and Neumann or Robin conditions on $\partial\Omega\cap\partial D$. The equivalent variational formulation

$$\lambda_1(\Omega;D)=\min\Big\{\int_\Omega|\nabla u|^2\,dx+k\int_{\partial D}u^2\,d\mathcal{H}^{d-1}\ :\ u\in H^1(D),\ u=0\text{ on }\partial\Omega\cap D,\ \|u\|_{L^2(\Omega)}=1\Big\}$$

reminds the classical drop problems, where the first eigenvalue replaces the total variation functional. We prove an existence result for general shape cost functionals and we show some qualitative properties of the optimal domains. The case of Dirichlet condition on a *fixed* part and of Neumann condition on the *free* part of the boundary is also considered.

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